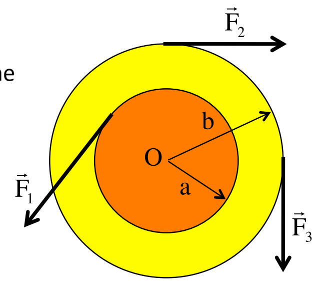
Problem 10.35

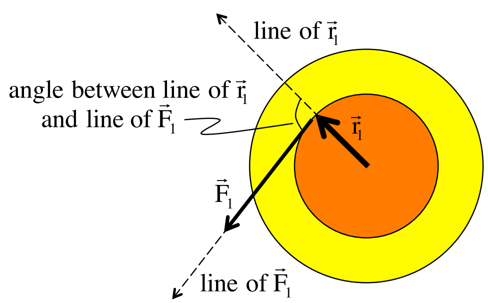
Determine the net torque about *O* produced by the forces shown.



The first thing to do when approaching a torque problem is to define a vector $\vec{r}$ that proceeds from the *point about which the torque* *is to be taken* *to the point where the* *force acts* (this is why free-body diagrams in rotational N.S.L. problems have to be accurately rendered—the placement of each force is significance in the torque calculation). Having said that, there are three ways to calculate a torque. As there are, conveniently, three forces here, I'll use a different approach for each.

Approach 1: Called *the definition approach,* it the simply the execution of the *cross product* $\vec{F}x\vec{d}$. That is, the magnitude of $\vec{F}$ times the magnitude of $\vec{d}$ times the sine of the angle between the line of the two vectors.

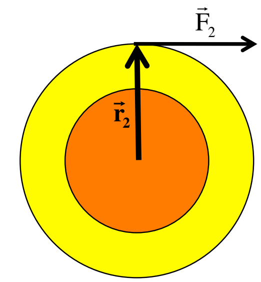
Defining $\vec{r}_1$ and the angle between $\vec{r}_1$ and $\vec{F}_1$ (see sketch), then using this approach on $\vec{F}_1$, we can write:



$$
\begin{aligned}
\left|\vec{\Gamma}_1\right| &= \vec{r}_1 x \vec{F}_1 \\
&= \left|\vec{F}_1\right|\left|\vec{r}\right|\sin\theta \\
&= \left|\vec{F}_1\right|(a)\sin 90^\circ \\
&= (12.0 \text{ N})(.100 \text{ m})\sin 90^\circ \\
&= 1.20 \text{ N}\bullet\text{m}
\end{aligned}
$$

There is a subtlety here. Force is a *vector*, which means *torque* must also be a vector. During class, I assume you were shown how to determine the direction of a torque (right-hand rule, or look to see if the force is motivating the body to angularly accelerate *clockwise*, at which time the torque would be *negative*, etc.) In any case, this torque is motivating the system to rotate *counterclockwise*, so the torque as a vector is *positive* and $\vec{\Gamma}_1 = +(1.20 \text{ N}\bullet\text{m})\hat{k}$.

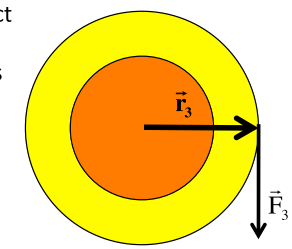
Approach 2: Called *the $r_\perp$ approach,* (or *the moment arm* approach), it is the product of the magnitude of the force times the magnitude of the component of $\vec{r}$ *perpendicular to the line of the force* (in most books, this perpendicular distance is called *the moment arm*). Note that in this case, the magnitude of the component of $\vec{r}$ *perpendicular to the force* is just "b," so:



$$
\begin{aligned}
\left|\vec{\Gamma}_2\right| &= \vec{r}_2 x \vec{F}_2 \\
&= \left|\vec{F}_2\right| r_{\perp,2} \\
&= \left|\vec{F}_2\right| b \\
&= (10.0 \text{ N})(.250 \text{ m}) \\
&= 2.50 \text{ N}\bullet\text{m}
\end{aligned}
$$

This torque is motivating the system to angularly accelerate *clockwise*, so the torque is negative and the torque as a vector is $\vec{\Gamma}_2 = -(2.50 \text{ N}\bullet\text{m})\hat{k}$.

Approach 3: Called *the $F_\perp$ approach*, this is the product of the magnitude of $\vec{r}$ times the magnitude of the component of $\vec{F}$ perpendicular to $\vec{r}$. Note that in this case, the component of $\vec{F}$ perpendicular to "b" is just $\left|\vec{F}_3\right|$:



$$
\begin{aligned}
\left|\vec{\Gamma}_3\right| &= \vec{r}_3 x \vec{F}_3 \\
&= \left|\vec{r}_3\right| F_{\perp,3} \\
&= b\left|\vec{F}_3\right| \\
&= (.250 \text{ m})(9.00 \text{ N}) \\
&= 2.25 \text{ N}\bullet\text{m}
\end{aligned}
$$

This torque is motivating the system to angularly accelerate *clockwise*, so the torque is negative and the torque as a vector is $\vec{\Gamma}_2 = -(2.50 \text{ N}\bullet\text{m})\hat{k}$.

The net torque is just the sum of these three, or:

$$
\begin{aligned}
\vec{\Gamma}_{net} &= \vec{\Gamma}_1 + \vec{\Gamma}_2 + \vec{\Gamma}_3 \\
&= (1.20 \text{ N}\bullet\text{m})\hat{k} + \left(-(2.50 \text{ N}\bullet\text{m})\hat{k}\right) + \left(-(2.25 \text{ N}\bullet\text{m})\hat{k}\right) \\
&= -(3.55 \text{ N}\bullet\text{m})\hat{k}
\end{aligned}
$$